ISSN 2229-5518

Fuzzification of Gamma-semigroups satisfying the identity the $x\alpha y\beta x = x\alpha y$

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Abstract— In this paper, we consider some properties and characterizations of fuzzy Γ - ideals and fuzzy bi Γ - ideals of Γ - semi groups and investigate some of their properties. We also characterize the properties related to Γ - semi groups and Fuzzy Γ - ideals using an identity $x\alpha y\beta x = x\alpha y$.

Index Terms— Γ - semi group, fuzzy bi Γ - ideal, fuzzy interior Γ - ideal, regular Γ - semi group,fuzzy subset,fuzzy sub Γ -semi group,fuzzy quasi Γ -ideal.

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1 INTRODUCTION

The fundamental concept of a fuzzy subset was introduced by L.A.Zadeh in 1965[4].The concept of fuzzy ideals in semi groups was introduced by N.Kuroki in 1979[2]. N.Kuroki [3] introduced fuzzy left (right) ideals, fuzzy bi ideals and fuzzy interior ideals. Some basic concepts of fuzzy algebra such as fuzzy left (right) ideals and fuzzy bi ideals in a fuzzy semi group were introduced by Dib [7] in 1994.D.R.Prince Williams and K.B.Latha introduced fuzzy Γ - ideal and fuzzy bi Γ - ideal [1].

Definition 1.1: A mapping $\mu : S \to [0,1]$ is called fuzzy subset of S and the compliment of a set μ, denoted by μ', is the fuzzy subset in S defined by $\mu' = 1 - \mu(x)$ *for all* $x \in S$. Let level set of a fuzzy subset μ of S is defined as $U(\mu,t) = \{x \in S / \mu(x) \geq t\}$. Note that Γ - semi group S can be considered as a fuzzy subset of itself and we write $S = C_S$ i.e. $S(x) = 1$ for all $x \in S$.

Definition 1.2: Let $S = \{x, y, z, ...\}$ and $\Gamma = \{\alpha, \beta, \gamma, ...\}$ be two non-empty sets then S is called a Γ-semi group if it satisfies (i) $x\gamma y \in S$ (ii) $(x\alpha y)\beta z = x\alpha(y\beta z)$ *for* $x, y, z \in S$ *and* $\alpha, \beta \in \Gamma$.

Definition 1.3: A fuzzy subset μ of S is called a fuzzy sub Γ- semi group of S if $\mu(x\alpha y) \geq \min\{\mu(x), \mu(y)\}$ for all $x, y \in S$ *and* $\alpha \in \Gamma$.

Definition 1.4: A fuzzy subset μ of S is called a fuzzy left (right) Γ - ideal of S if

$$\mu(x\alpha y) \geq \mu(y) \ (\ \mu(x\alpha y) \geq \mu(x)\ )$$

for all $x, y \in S$ *and* $\alpha \in \Gamma$.

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Definition 1.5: A fuzzy subset μ of S is called a fuzzy Γ - ideal of S if it is both fuzzy left Γ - ideal and fuzzy right Γ - ideal of S.

Definition 1.6: A fuzzy sub Γ - semi group μ of S is called a fuzzy bi Γ - ideal of S if $\mu(x\alpha y\beta z) \geq \min\{\mu(x), \mu(z)\}$ for all $x, y, z \in S$ *and* $\alpha, \beta \in \Gamma$.

Definition 1.7: A fuzzy sub Γ - semi group μ of S is called a fuzzy interior Γ - ideal of S if $\mu(x\alpha y\beta z) \geq \mu(y)$ for all $x, y, z \in S$ *and* $\alpha, \beta \in \Gamma$.

Definition 1.8: Let μ_1 and μ_2 be two fuzzy subsets of Γ- semi group S. Then $\mu_1 \cap \mu_2$ *and* $\mu_1 \cup \mu_2$ are defined by $(\mu_1 \cap \mu_2)(a) = \min\{\mu_1(a), \mu_2(a)\}$ and $(\mu_1 \cup \mu_2)(a) = \max\{\mu_1(a), \mu_2(a)\}$.
We denote $\wedge$-minimum or infimum and $\vee$- maximum or suprimum then

$$(\mu_1 \cap \mu_2)(a) = \mu_1(a) \wedge \mu_2(a),$$
$$(\mu_1 \cup \mu_2)(a) = \mu_1(a) \vee \mu_2(a).$$

Definition 1.9: Let μ_1 and μ_2 be any two fuzzy subsets of a Γ- semi group S. Then their fuzzy product $\mu_1 \circ \mu_2$ is defined by

$\mu_1 \circ \mu_2$ (a)=Sup{ $\mu_1(x) \wedge \mu_2(y)$ } if $a = x\alpha y$ *for* $x, y \in S$ *and* $\alpha \in \Gamma$ and
$\mu_1 \circ \mu_2$ (a)=0 othervise.

Definition 1.10: A fuzzy sub Γ- semi group μ of S is called a fuzzy bi Γ-ideal of a Γ- semi group S if $\mu(x\alpha y\beta z) \geq \mu(x) \wedge \mu(z)\}$ for all $x, y, z \in S$ *and* $\alpha, \beta \in \Gamma$.

Definition 1.11: A fuzzy subset μ of a Γ- semi group S is a fuzzy quasi Γ- ideal of S if

$$(\mu \circ S) \cap (S \circ \mu) \subseteq \mu$$

ISSN 2229-5518

2. Main Results:

Theorem.2.1: Let S be a regular $\Gamma - semigroup$ and S satisfy the identity $x\alpha y\beta x = x\alpha y, \forall x, y \in S \; and \; \alpha, \beta \in \Gamma$ then for any non-empty fuzzy set μ of S we have,

$i)\mu(x\alpha y) = \mu(x)$ $ii)\mu(x\alpha x) = \mu(x)$

$iii)\mu((x\alpha)^n x) = \mu(x)$, $n \in N$.

Proof:-Since S is a regular $\Gamma - semigroup$, we have for any $x \in S \Rightarrow x\alpha y\beta x = x$ for some $y \in S \; and \; \alpha, \beta \in \Gamma$.

i) Consider $\mu(x\alpha y) = \mu((x\alpha y\beta x)\alpha y)$ [$\because x$ *is regular in* S]

$= \mu(x\alpha(y\beta x\alpha y))$ [Associativity in S]

$= \mu(x\alpha y\beta x)$ [using the given identity]

$= \mu(x)$. [$\because x$ *is regular in* S]

$\therefore \mu(x\alpha y) = \mu(x).$

ii) From (i) we have $\mu(x\alpha y) = \mu(x).$

replace y by x we get ,

$\mu(x\alpha x) = \mu(x).$ hence proved

iii) We see that $\mu(x\alpha x) = \mu(x).$

$\mu(x\alpha x\alpha x) = \mu(x\alpha y)$ *where* $x\alpha x = y \in S$

$= \mu(x)$ From (i).

$\therefore \mu((x\alpha)^2 x) = \mu(x)$.

Again $\mu(x\alpha(x\alpha)^2 x) = \mu(x\alpha y)$ *where* $(x\alpha)^2 x = y \in S$

$= \mu(x)$ From (i).

$\therefore \mu((x\alpha)^3 x) = \mu(x)$.

In general $\mu((x\alpha)^n x) = \mu(x)$, $n \in N$.

Theorem 2.2: Let μ be a *fuzzy bi* $\Gamma - ideal$ in a $\Gamma - semigroup$ S and S satisfy the identity $x\alpha y\beta x = x\alpha y, \forall x, y \in S \; and \; \alpha, \beta \in \Gamma$, then μ is a *fuzzy right* $\Gamma - ideal$ in S.

Proof:- Let μ be a *fuzzy bi* $\Gamma - ideal$ in a $\Gamma - semigroup$ S ,then

$$\mu(x\alpha y\beta z) \geq \min\{\mu(x), \mu(z)\}, \forall x, y, z \in S \; and \; \alpha, \beta \in \Gamma \quad (1)$$

And is a fuzzy sub $\Gamma - semigroup$ of S.

Given S satisfy the identity $x\alpha y\beta x = x\alpha y, \forall x, y \in S \; and \; \alpha, \beta \in \Gamma$

We have to show that μ is a *fuzzy right* $\Gamma - ideal$ of S.

i.e. $\mu(x\alpha y) \geq \mu(x), \forall x, y \in S \; and \; \alpha \in \Gamma$.

Consider $\mu(x\alpha y) = \mu(x\alpha y\beta x)$ [using the given identity]

$\geq \min\{\mu(x), \mu(x)\}$ [using (1)]

$\therefore \mu(x\alpha y) \geq \mu(x)$.

$\therefore \mu$ is a *fuzzy right* $\Gamma - ideal$ in S.

Theorem 2.3: Let μ be a fuzzy interior Γ- ideal in a $\Gamma - semigroup$ S and S satisfy the identity $x\alpha y\beta x = x\alpha y, \forall x, y \in S \; and \; \alpha, \beta \in \Gamma$,then μ is a *fuzzy left* $\Gamma - ideal$ in S.

Proof:- Let μ be a *fuzzy interior* $\Gamma - ideal$ in a $\Gamma - semigroup$ S

$\therefore \mu(x\alpha y\beta z) \geq \mu(y), \forall x, y, z \in S \; and \; \alpha, \beta \in \Gamma$

Consider $\mu(x\alpha y) = \mu(x\alpha y\beta x)$ [using the given identity]

$\geq \mu(y)$ [$\because \mu$ is a *fuzzy interior* $\Gamma - ideal$]

$\therefore \mu(x\alpha y) \geq \mu(y), \forall x, y \in S \; and \; \alpha \in \Gamma$.

$\therefore \mu$ is a *fuzzy left* $\Gamma - ideal$ of S.

Theorem 2.4:-Let μ be a *fuzzy left* $\Gamma - ideal$ in a $\Gamma - semigroup$ S and S satisfy the identity $x\alpha y\beta x = x\alpha y, \forall x, y \in S \; and \; \alpha, \beta \in \Gamma$,then μ is a *fuzzy sub* $\Gamma - semigroup$ of S.

Proof: Let μ be a *fuzzy left* $\Gamma - ideal$ in a $\Gamma - semigroup$ S

$\therefore \mu(x\alpha y) \geq \mu(y), \forall x, y \in S \; and \; \alpha \in \Gamma$ (2)

To prove that μ is a fuzzy sub gamma-semi group.

i.e. $\mu(x\alpha y) \geq \min\{\mu(x), \mu(y)\}, \forall x, y \in S \; and \; \alpha \in \Gamma$

Consider $\mu(x\alpha y) = \mu(x\alpha y\beta x)$ [using the given identity]

$= \mu((x\alpha y)\beta x)$ [using associativity in S]

$\therefore \mu(x\alpha y) \geq \mu(x)$. [using (2)]

$\Rightarrow \mu(x\alpha y) \wedge \mu(x\alpha y) \geq \mu(x) \wedge \mu(x\alpha y)$

$\geq \mu(x) \wedge \mu(y)$ [μ is a *fuzzy left* $\Gamma - ideal$]

$\Rightarrow \mu(x\alpha y) \geq \mu(x) \wedge \mu(y)$

$\therefore \mu(x\alpha y) \geq \min\{\mu(x), \mu(y)\}, \forall x, y \in S \; and \; \alpha \in \Gamma$

$\therefore \mu$ is a *fuzzy sub* $\Gamma - semigroup$ of S.

Theorem 2.5: Let μ be a *fuzzy right* $\Gamma - ideal$ of a commutative $\Gamma - semigroup$ S, then μ is a *fuzzy sub* $\Gamma - semigroup$ of S.

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ISSN 2229-5518

Proof: Let μ be a $fuzzy\ right\ \Gamma-ideal$ of a commutative $\Gamma-semigroup\ S$,then

$$\mu(x\alpha y) \geq \mu(x),\ \forall x, y \in S\ and\ \alpha \in \Gamma$$

To prove that μ is a $fuzzy\ sub\ \Gamma-semigroup$ of S .

We see that $\mu(x\alpha y) \geq \mu(x),\ \forall x, y \in S\ and\ \alpha \in \Gamma$

$$\therefore\ \mu(x\alpha y) \wedge \mu(y\alpha x) \geq \mu(x) \wedge \mu(y\alpha x)$$
$$\geq \mu(x) \wedge \mu(y)$$
$$\because\ \mu\ is\ fuzzy\ right\ \Gamma-ideal\ of\ S$$
$$\therefore\ \mu(x\alpha y) \wedge \mu(x\alpha y) \geq \mu(x) \wedge \mu(y)$$
$$\because S\ is\ a\ commutative$$
$$\therefore\ \mu(x\alpha y) \geq \mu(x) \wedge \mu(y)$$

$$\therefore\ \mu(x\alpha y) \geq \min\{\mu(x), \mu(y)\}, \forall x, y \in S\ and\ \alpha \in \Gamma$$
$$\therefore\ \mu \text{ is a } fuzzy\ sub\ \Gamma-semigroup \text{ of } S .$$

Theorem 2.6: Let μ be a $fuzzy\ left\ \Gamma-ideal$ in a $\Gamma-semigroup\ S$ and S satisfy the identity $x\alpha y\beta x = x\alpha y, \forall x, y \in S\ and\ \alpha, \beta \in \Gamma$,then $\mu(x\alpha y) = \mu(y\alpha x)$.

Proof:- Let μ be a $fuzzy\ left\ \Gamma-ideal$ in a $\Gamma-semigroup\ S$

$$\therefore\ \mu(x\alpha y) \geq \mu(y), \forall x, y \in S\ and\ \alpha \in \Gamma$$

To prove that $\mu(x\alpha y) = \mu(y\alpha x)$ we shall prove that $\mu(x\alpha y) \leq \mu(y\alpha x)$ and $\mu(x\alpha y) \geq \mu(y\alpha x)$

Now consider $\mu(x\alpha y) = \mu(x\alpha y\alpha x)$

[using the identity]

$= \mu(x\alpha(y\alpha x))$

[using associativity in S]

$\geq \mu(y\alpha x)$

$\because\ \mu$ be a $fuzzy\ left\ \Gamma-ideal$

$$\therefore\ \mu(x\alpha y) \geq \mu(y\alpha x) \qquad (3)$$

Similarly consider $\mu(y\alpha x) = \mu(y\alpha x\alpha y)$

[using the identity]

$= \mu(y\alpha(x\alpha y))$

[using associativity in S]

$\geq \mu(x\alpha y)$

$\because\ \mu$ be a $fuzzy\ left\ \Gamma-ideal$

$$\therefore\ \mu(y\alpha x) \geq \mu(x\alpha y) \qquad (4).$$

$\therefore$ from (3) and (4) we have $\mu(x\alpha y) = \mu(y\alpha x)$.

Theorem 2.7: Let μ be a $fuzzy\ left\ \Gamma-ideal$ of a $\Gamma-semigroup\ S$ and S satisfy the identity $x\alpha y\beta x = x\alpha y, \forall x, y \in S\ and\ \alpha, \beta \in \Gamma$,then $\mu \circ \mu \leq \mu$.

Proof:- Let μ be a $fuzzy\ left\ \Gamma-ideal$ of a $\Gamma-semigroup\ S$

Given S satisfy the identity

$x\alpha y\beta x = x\alpha y, \forall x, y \in S\ and\ \alpha, \beta \in \Gamma$

Then from theorem 2.4

$$\mu(x\alpha y) \geq \min\{\mu(x), \mu(y)\}, \forall x, y \in S\ and\ \alpha \in \Gamma \quad (5)$$

Now

$$\mu \circ \mu(x) = V\{\mu(y) \wedge \mu(z).\} where\ x = y\alpha z\ and\ y, z \in S$$
$$\leq V\ \ \mu(y\alpha z) \quad \text{[from (5)]}$$
$$\mu \circ \mu(x) \leq \mu(y\alpha z)$$
$$\mu \circ \mu(x) \leq \mu(x) \qquad \because\ x = y\alpha z]$$
$$\therefore\ \ \mu \circ \mu \leq \mu .$$

Theorem 2.8: Let μ be a $fuzzy\ right\ \Gamma-ideal$ of a commutative $\Gamma-semigroup\ S$ then $\mu \circ \mu \leq \mu$.

Proof: Let μ be a $fuzzy\ right\ \Gamma-ideal$ of a commutative $\Gamma-semigroup\ S$.

$\therefore$ from theorem 2.5

$$\mu(x\alpha y) \geq \min\{\mu(x), \mu(y)\}, \forall x, y \in S\ and\ \alpha \in \Gamma \quad (6)$$
$$\mu \circ \mu(x) = V\{\mu(y) \wedge \mu(z).\} where\ x = y\alpha z\ and\ y, z \in S$$
$$\leq V\ \ \mu(y\alpha z) \quad \text{[from (6)]}$$
$$\mu \circ \mu(x) \leq \mu(y\alpha z)$$
$$\mu \circ \mu(x) \leq \mu(x) \qquad \because\ x = y\alpha z]$$
$$\therefore\ \ \mu \circ \mu \leq \mu$$

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